Math 3325: Transitions to Advanced Mathematics

This is a course guideline. Students should contact instructor for the updated information on current course syllabus, textbooks, and course content

Purpose:

This course is an introduction to proofs and the abstract approach that characterizes upper level mathematics courses. It serves as a transition into advanced mathematics, and should be taken after the initial calculus sequence and before (or concurrently with) mid-level mathematics courses. The goal is to give students the skills and techniques that they will need as they study any type of advanced mathematics, whether it be in pure mathematics, applied mathematics, or application-oriented courses. In particular, this course covers topics that are ubiquitous throughout mathematics (e.g. logic, sets, functions, relations) and helps prepare students for classes such as Real Analysis, Abstract Algebra, and Advanced Linear Algebra, that are required for majors and minors.

A major objective of the course will be to teach students how to read, write, and understand proofs. Throughout the course students will be exposed to the notation, language, and methods used by mathematicians, and will gain practice using these in their own proofs. In addition, great emphasis will be placed on writing and communication.

Prerequisites: Calculus I and Calculus II.

Suggested Texts:

- A Transition to Advanced Mathematics, by Douglas Smith, Maurice Eggen, and Richard St. Andre

- Mathematical Proofs: A Transition to Advanced Mathematics, by Gary Chartrand, Albert D. Polimeni, and Ping Zhang

- Transition to Higher Mathematics: Structure and Proof by Bob Dumas and John McCarthy

- Proofs and Fundamentals: A First Course in Abstract Mathematics by Ethan D. Bloch

Course Outline: Items I–VII and VIIIB are expected to be covered. Avoiding excessive formality (like spending much time on truth tables, which many students meet in other courses) will help with covering the syllabus. VIIIC and D below are optional, however choosing to cover some easy examples of epsilon-delta proofs in R will greatly benefit students going on to 3333 (for example all our Math Majors).

I. Introduction to Advanced Mathematics

A. What is Mathematics
B. Inductive vs. Deductive Reasoning
C. What is a Proof?

II. Logic and proofs

A. Truth Tables, Conditionals ($P \Rightarrow Q$), and Biconditionals ($P \Leftrightarrow Q$)
B. Negation, Converse, and Contrapositive
C. Existential and Universal Quantifiers ($\forall, \exists, \exists!$)
D. Proof Techniques (Contrapositive, Contradiction, Induction), Counterexamples, and Proving Statements with Quantifiers
E. Writing Proofs (Conventions, Notation, and Style)

III. Set Theory and its Axioms

A. Sets and Set Notation, the Empty Set, the Power Set
B. Union, Intersection, Complement, Subsets
C. Proving sets are equal
D. Axioms of Naïve Set Theory
E. The Axiom of Choice

IV. The Natural Numbers

A. Natural Numbers and the Peano Postulates
B. Principle of Mathematical Induction, Principle of Strong Mathematical Induction, Well-Ordering Principle
C. Divisibility, Greatest Common Divisors, Least Common Multiples, and the Division Algorithm

V. Relations

A. Cartesian Products and Relations
B. Equivalence Relations and Partitions
C. Partial Orderings, Least Upper Bounds, and Greatest Lower Bounds

VI. Functions

A. Definition of a Function, Domains and Codomains
B. Composition and Inverses
C. Verifying a Function is Well-Defined
D. Injective, Surjective, and Bijective Functions
E. Invertibility of Functions

VII. Cardinality

A. One-to-One Correspondences and Set Equivalence
B. Cardinality of Finite Sets (and the Pigeon-Hole Principle)
C. Cardinality of Infinite Sets
D. Denumerable, Countable, and Uncountable Sets
E. The Partial Ordering of the Cardinal Numbers, and the Cantor-Schröder-Bernstein Theorem

VIII. Basic Properties of the Real Numbers

A. Axioms of the Real Numbers
B. Completeness of the Real Numbers, Supremums, and Infimums
C. Limits of Sequences
D. (ε,δ)-definition of Limits and Continuity